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Nonlinear Analysis of Lateral Loading During Taxiway Turns

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A general approach to assess an aircraft's performance during taxiway maneuvers across the range of its operation is presented. The main motivation for this work is to evaluate the suitability of the existing Federal Aviation Regulation for lateral loads experienced during turning maneuvers. To this end, operating regions are defined in terms of parameters specifying the approach velocity and steering input for a generic turn that is representative of pilot practice. The limits of the operating regions represent the extremes of the aircraft's operation during turning as determined by the maximal lateral loading conditions identified in published studies of instrumented in-service passenger aircraft. The performance of the turn can be assessed over the entire operational range in terms of the actual loads experienced at individual landing gears. Recent studies by the Federal Aviation Administration of instrumented aircraft have been limited to investigating the lateral loads experienced at the aircraft's center-of-gravity position. The results presented here show that this information is insufficient to predict the actual loads experienced by individual landing gears, especially for the nose gear, which is found to experience considerably higher lateral loads than predicted by the corresponding loads at center of gravity. These findings are shown to be robust with respect to changes in the aircraft's mass and the criterion used to define the limits of the operating regions.

Nomenclature

D_{CG}	=	maximum deviation of center-of-gravity position from turn centerline, m
D_{NLG}	=	maximum deviation of nose landing gear from turn centerline, m
L	=	longitudinal lag, m
N_{CG}	=	scaled lateral load factor at center of gravity, g
N_{ILG}	=	scaled lateral load at inner landing gear, no units
N_{NLG}	=	scaled lateral load at nose landing gear, no units
N_{OLG}	=	scaled lateral load at outer landing gear, no units
R	=	turn radius, m
t_{fin}	=	steer ramp required time, m
V_{init}	=	initial velocity, m/s
V_{loss}	=	velocity lost during turn, %
X	=	lateral displacement, m
Y	=	longitudinal displacement, m
δ_{fin}	=	target steering angle, deg
$\delta_N(t)$	=	steering angle at time t, deg
δ_{rate}	=	maximum steer rate, deg/s

I. Introduction

THE landing gears of commercial aircraft are subject to substantial lateral loads during taxiing. For example, when exiting the runway at relatively high velocity, it is necessary for the tires to generate sufficiently large lateral forces to complete the maneuver. There is a tradeoff between increasing the structural strength of a landing gear to accommodate larger loads and the associated weight penalty. Therefore, it is important to identify the maximal lateral load values and the conditions under which they occur. This information can be used to assess the suitability of current regulations, to inform the design of future aircraft and to improve operational practice.

The regulation imposed by the Federal Aviation Administration (FAA) on the lateral loads experienced during turning for the certification of new civil aircraft is specified in Federal Aviation Regulation (FAR) 25.495. Termed the 0.5 g lateral acceleration criterion, the regulation has two key parts with regards to the lateral loads experienced at the aircraft's center-of-gravity (CG) position. Firstly, the limit loads during steady turning must not exceed 0.5 g laterally. Secondly, each gear must structurally be able to withstand half of its maximum static vertical load applied laterally. In the regulation there is an inherent assumption that the limiting lateral load of 0.5 g is evenly distributed between the aircraft's landing gears. The FAA have expressed concerns about the suitability of this regulation for the certification of modern passenger aircraft; in particular, the regulation is perceived to be too conservative for larger aircraft that have more than two main landing gears [1,2]. With the aim of evaluating the existing regulation, the FAA have instrumented in-service aircraft and carried out a series of extensive studies to determine the actual lateral loads experienced during ground maneuvers [1–4]; these studies are discussed below.

Because of considerations of both cost and safety in performing specific ground tests, it is advantageous to use computer modelling to extend and complement the FAA studies. Indeed, computer simulation has been used previously to study the dynamics of aircraft on the ground; examples are a series of investigations by Klyde et al. that use a combination of mathematical modelling and ground tests [5–7]. Another study by Khapane investigates asymmetric landing and ground maneuvers with a model implemented in the multibody systems package SIMPACK, which includes nonlinear effects [8]. A multibody systems approach has been used extensively in the study of vehicle dynamics [9,10]. Nonlinearities play a significant role in the dynamics of aircraft on the ground, especially when studying behavior close to or beyond the limits of normal operation. A previous study by the authors [11] used an industry-tested, nonlinear model implemented in the multibody systems package SimMechanics to

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identify regions of safe operation in terms of the control inputs of the aircraft. In contrast to existing work, the model was analyzed with tools from nonlinear dynamics, specifically, a bifurcation analysis was performed. The focus of the work was a steady-state analysis in which stability boundaries are detected and followed under variation of parameters. In a more recent study by the authors [12] a fully parametrized mathematical model was developed and validated against the existing SimMechanics model. The advantage of this aircraft model, which is also used here, is that it allows direct access to all system states and parameters. The mathematical model was previously used to investigate changes in the stability limits with respect to variation of operational parameters, such as the aircraft's mass and CG position.

The specific aim of this work is to investigate lateral loading during ground maneuvers in order to assess the suitability of the regulation described above. First of all, it is necessary to give further details of the existing investigations carried out by the FAA. Reference [3] provides a statistical analysis of flight and loads data from a specific in-service aircraft recorded over the course of more than 30,000 flight hours. Included in the report is relevant usage data; for example, cumulative occurrences of lateral load factor recorded during different phases of the aircraft's ground operations. The later study [4] summarizes and compares such data recorded from a range of different size aircraft. The more recent study [1] focuses specifically on lateral loads during ground maneuvers and makes improvements in terms of the presentation of the data. In particular, the data is organized by aircraft model to allow comparison between the lateral loads experienced during different ground phases. Figure 1, reproduced from [1], shows cumulative occurrences of lateral load per flight scaled in terms of the operating weight for a typical medium-sized passenger aircraft. The data is broken up into different phases of the aircraft's ground operation. For the purposes of the present investigation, the relevant data is that recorded during the taxi-in, taxi-out, landing roll and runway turnoff phases. The maximal lateral load factor, scaled by aircraft weight, recorded during the taxi-out phase is 0.2 g, during the taxi-in phase it is 0.19 g, during the landing roll phase it is 0.25 g, and during the turnoff phase it is 0.24 g. For convenience, these phases are grouped together as follows. The taxi-out and taxi-in phases are grouped together, and denoted the taxi phase, because they consist of similar maneuvers; the overall maximal lateral load factor for the taxi phase is 0.2 g. The landing roll and turnoff phases are grouped together and denoted the runway turnoff phase. The landing roll, which immediately precedes the runway turnoff, is included in order to capture loads recorded as the turnoff maneuver is initiated. The overall maximal lateral load factor for the runway turnoff phase is 0.25 g. Larger loads occur during runway turnoff due to greater velocities immediately after landing. The data from this study suggests that the regulation limit for the lateral load factor is conservative. The effect of asymmetric loading between the landing gears is not taken into account in [1] and information with regards to the conditions under which specific lateral load values are attained is limited. The most recent study [2] presents limited ground test data recorded from an instrumented large commercial aircraft with more than two main landing gears. The significance of asymmetric loading between the main landing gears is investigated, but no information is provided about the nose landing gear (NLG). For specificity, in the remainder of the paper, the results presented here are compared with usage data from [3] and the scaled loads data for a specific medium-sized passenger aircraft from [1].

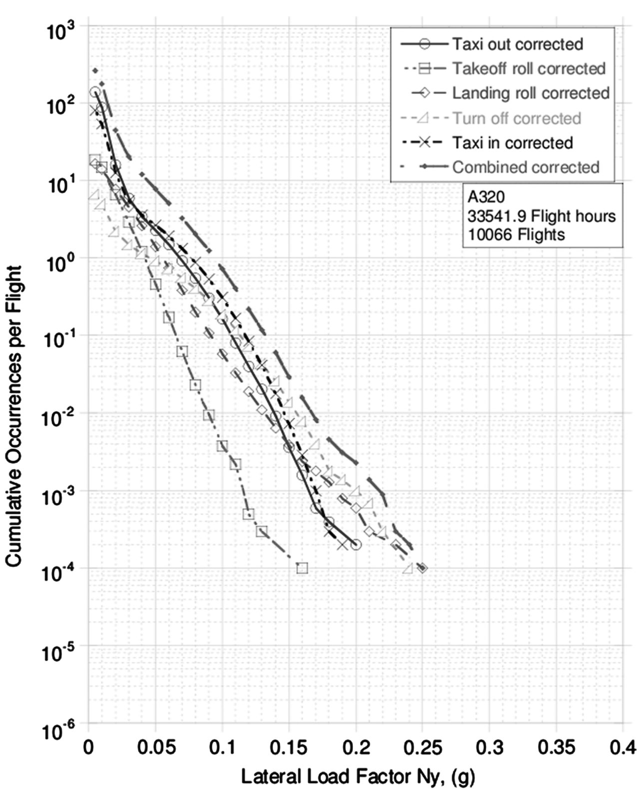

Fig. 1 FAA instrumentation data (reproduction of figure A-27 from [1]) showing cumulative occurrences per flight of lateral load factor, corrected/scaled by operating weight corrected, separated into different ground phases.

A general approach to evaluate an aircraft's performance across an entire operating region for specific turning maneuvers is presented. The focus is on two types of turning maneuver: a runway turnoff maneuver that corresponds to the runway turnoff phase data, and a taxiway-to-taxiway transition that corresponds to the taxi phase data. The maximal lateral load factors for the two ground phases, identified above in the FAA report data, are considered to represent a practical upper bound that is not exceeded for the respective turning maneuvers. Because of the large size of the data sets represented by the FAA studies, it is reasoned that the limit lateral load factors are not surpassed in the day-to-day operation of the aircraft. With the aim of studying the actual landing gear loads at the limits of operation, a parametrized turn is defined in terms of the turn approach velocity and the steering input during the turn. Taking into account the runway and taxiway geometry, the parametrized turn is linked directly to the two maneuvers under consideration. Parameter values at which the limit lateral load cases occur are identified; based on this information operating regions are defined. The actual gear loads are found at the limits of the operating regions and, therefore, at the limit of the aircraft's operation. The maximal gear loads are found for the two types of maneuver and two different mass cases (operating weights). The effect of asymmetric lateral loading between all the landing gears is studied and the effect of different overall mass on the actual gear loads experienced is investigated. It is found that the lateral load factor at CG is sufficient for the prediction of the maximal loads at the main landing gears, but not sufficient for the prediction of loads at the nose gear. Furthermore, it is found that the loads at the nose gear are significantly underestimated by the lateral load factor at CG. The results suggest that, for the specific aircraft under consideration, the existing regulation is too conservative for the main landing gears, but this is not necessarily the case for the nose gear. Other regulations, for example the towing regulation FAR 25.509, may account for larger lateral loads on the nose gear; however, the result is still important with respect to fatigue loading. An advantage of the general approach presented is that the limits of operation can be defined in terms of any user specified criteria. As an example, a similar study is performed with operating regions defined in terms of a criterion that ensures efficiency of the maneuvers. Overall, the approach presented here gives insights into the conditions under which the maximal loading cases identified in the FAA data occur, and extended information about actual gear loads at the limits of operation. It is demonstrated that the approach used here is not limited to the study of the extremes of operation. Furthermore, although the focus here is on the loads experienced at individual landing gears, the approach is applicable for the study of any aircraft states of interest.

This study uses the fully parametrized mathematical model from [12] of a typical medium-sized, single-aisle passenger aircraft implemented in MATLAB. The aircraft is modeled as a tricycle with the rigid airframe having three translational and three rotational degrees of freedom. The equations of motion were obtained via the balancing of forces and moments in each degree of freedom. Nonlinear effects are included in the tire model [13], depending on tire load and slip angle, and in the aerodynamic model, depending on velocity, angle of attack and sideslip angle of the airframe [14]. In addition to these nonlinearities in the models of local components, the equations of motion themselves are inherently nonlinear due to the dependence of the tire forces and aerodynamic forces on the system states. A full description of the model and its validation against an established industry-tested SimMechanics model are provided in [12]. The advantage of using a low-order mathematical model is that it is computationally inexpensive to perform a larger number of simulations. Furthermore, the model allows direct access to component forces such as the lateral forces acting on individual landing gears. This allows for an analysis of the load distribution between individual landing gears and a comparison of this information with loads experienced at the aircraft's CG position.

In this paper two mass cases are defined that allow for convenient comparison with the loads data presented in [1]. In the FAA study, the recorded lateral loads are scaled in terms of the aircraft's maximum landing weight (MLW) of 64,560 kg. In this paper the loads are scaled in the same way. A comparison of the load values reported in [1] before and after this scaling shows that the maximal lateral load cases correspond to a mass value of approximately 0.75× MLW = 48420 kg, which is close to the minimal operating weight recorded in [3]. Accordingly, the two cases used here are a heavy operating case at the MLW and a light operating case at 0.75× MLW. To compare our results directly with the scaled loads data presented in [1], the lateral load factor N_{CG} is defined as maximal lateral load N_y recorded at the CG position during the turn, scaled by the ratio of the operating weight with the MLW. So, the lateral load factor $N_{\mathrm{CG}} = \max(N_y) \times \frac{\mathrm{OW}}{\mathrm{MLW}}$. For all of the results in this paper a forward CG position of 17% of the aircraft's mean aerodynamic chord is used. In the results sections of this paper, the loads experienced at individual landing gears are discussed. For consistency, the loads at the landing gears are scaled to allow direct comparison with loads at the CG position. The loads on the individual gears are normalized with respect to maximum vertical load on the gear under static loading. For the NLG this corresponds to a heavy aircraft (at MLW) with a forward CG position and the vertical load under static loading is 92 kN. For the main landing gears this corresponds to a heavy aircraft with an aft CG position and the vertical load is 300 kN. In the results presented here, it is assumed that the aircraft always turns to the right and, therefore, the outer landing gear (OLG) is defined as the left-hand gear and the inner landing gear (ILG) is defined as the right-hand gear. The lateral gear load N_{NLG}, N_{ILG} or N_{OLG} refers to the maximal load recorded at the respective landing gear during the turn, divided by the static load values given above; these quantities are defined in a similar fashion to the lateral load factor N_{CG}. For example, a lateral NLG load of $N_{\mathrm{NLG}} = 0.5$ corresponds to an actual load at the NLG of 0.5×92 kN = 46 kN.

The results in this paper are organized as follows. In Sec. II the parametrized turn is described. In Sec. III operational regions are found in terms of the parameters for different types of turn. In Sec. IV the maximal lateral loads at the limits of the operational regions are determined. New operating regions are defined in Sec. V with respect to the efficiency of turns. Conclusions drawn from the results are presented in Sec. VI.

II. Generic Parametrized Turn

In this section a parametrized turn appropriate for the study of lateral loading during taxi maneuvers is defined. The aim is to characterize a general turning procedure that is representative of pilot practice. Furthermore, for any given taxiway maneuver, there are a number of ways to perform that maneuver. Dependent on factors such as the velocity when entering a turn and steering characteristics, the lateral loads experienced during the maneuver vary significantly. The various factors discussed here are taken into account in the definition of a parametrized turn.

Typically, when the aircraft is approaching a turn on a straight section of taxiway the brakes are applied to achieve a desired velocity before entering the turn. After braking the turn is initiated with the application of steering. The velocity before entering the turn is represented here by the parameter V_{init} (with units m/s). In the simulations the initial condition describes the aircraft travelling in a straight line with the thrust set so that it is at equilibrium with fixed velocity V_{init}. From the initial condition the turn is initiated with the application of the steering; the steering angle is ramped up from 0° to a target value denoted δ_{fin} (given in degrees) which is taken as the second parameter to characterize the turn. The idealized steering profile used here is shown in Fig. 2a; it is represented by the function

$$\delta_N(t) = \frac{\delta_{\mathrm{fin}}}{2}\left[1 + \tanh\left(\frac{\delta_{\mathrm{rate}}}{\delta_{\mathrm{fin}}}(2t - t_{\mathrm{fin}})\right)\right]$$

where $\delta_N(t)$ is the steering angle applied at the nose gear at time t; furthermore, δ_{rate} is the fixed maximum steering rate and $t_{\mathrm{fin}} = \frac{3\delta_{\mathrm{fin}}}{\delta_{\mathrm{rate}}}$ is the required time to ramp up the steering such that $\delta_N(t_{\mathrm{fin}}) = \delta_{\mathrm{fin}}$. The realistic value of $\delta_{\mathrm{rate}} = 12$ deg/s is used and the maximum rate is achieved at $t = \frac{t_{\mathrm{fin}}}{2}$.

In the remainder of this section the properties of the parametrized turn are studied for different values of the target steering angle δ_{fin} and initial velocity V_{init}. In this section the resulting trajectories are studied independently of the taxiway geometry and in Sec. III the trajectories are related directly to taxiway geometry. Each simulation gives a trajectory describing the motion of the aircraft over the (X, Y) ground-plane and associated time history data for the system states; the coordinates X and Y are given in metres. It is straightforward to extract detailed information from the model, such as the forces experienced at the ground–tire interactions.

A previous study by the authors identified two possible types of behavior; when the aircraft makes a turn it can either converge to a

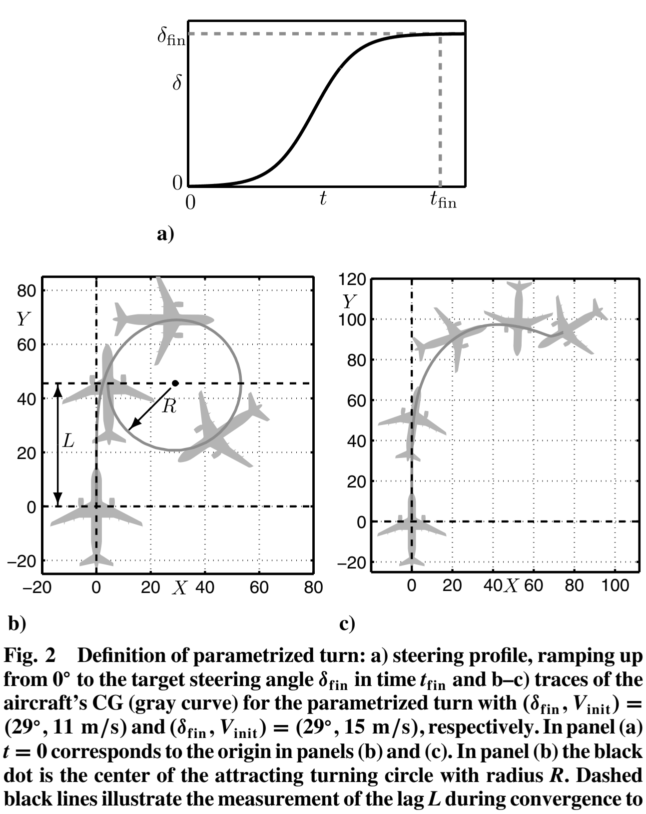

Fig. 2 Definition of parametrized turn: a) steering profile, ramping up from 0° to the target steering angle δ_{fin} in time t_{fin} and b–c) traces of the aircraft's CG (gray curve) for the parametrized turn with $(\delta_{\mathrm{fin}}, V_{\mathrm{init}}) = (29°, 11\ \mathrm{m/s})$ and $(\delta_{\mathrm{fin}}, V_{\mathrm{init}}) = (29°, 15\ \mathrm{m/s})$, respectively. In panel (a) $t = 0$ corresponds to the origin in panels (b) and (c). In panel (b) the black dot is the center of the attracting turning circle with radius R. Dashed black lines illustrate the measurement of the lag L during convergence to the turning circle. In panel (c) the aircraft loses lateral stability and at the final point in the trajectory it is stationary.

stable turning circle solution or, if the maneuver is too aggressive, there is a loss of lateral stability [11]. Figure 2b and 2c are two example trajectories; plotted is a trace of the aircraft's CG position (gray curve) over the (X, Y) ground plane with markers plotted to scale at equally spaced time intervals that indicate the aircraft's orientation along the trajectory. Figure 2b shows a trajectory computed for $(\delta_{\text{fin}}, V_{\text{init}}) = (29°, 11\ \text{m/s})$, for which the aircraft converges to a stable turning circle after a transient period. Illustrated are two quantities that describe the geometry of a stable trajectory. The radius of the turning circle to which the aircraft converges is denoted R (with units m). The longitudinal distance travelled from the initiation of the steering ramp at $t = 0$ to the point where the center of the turning circle is passed is referred to as the approach lag; it is denoted L (with units m). For illustrative purposes, the parameter values of δ_{fin} and V_{init} for the trajectory shown in Fig. 2b were chosen to exaggerate L. In general, when δ_{fin} is increased the radius R decreases as the aircraft follows tighter turns; when either δ_{fin} or V_{init} is increased the lag L increases as there is a longer delay before the aircraft makes the turn. Figure 2c shows a maneuver computed for $(\delta_{\text{fin}}, V_{\text{init}}) = (29°, 15\ \text{m/s})$; with this greater initial velocity the aircraft loses lateral stability. This laterally unstable behavior has been studied at length in reference [11]; here the boundary between the two types of behavior is identified but the main focus is on stable turning.

The implementation of a relatively low-order model in MATLAB allows for the computation of large numbers of model simulations across a two-dimensional parameter space at low computational cost. A 200×200 grid of values for the parameters δ_{fin} and V_{init} is taken over the ranges $\delta_{\text{fin}} \in (2°, 25°)$ and $V_{\text{init}}(5, 25)$ m/s. The velocity range is chosen to cover values representing relatively low-speed turns up to values in excess of the limits of operation. The maximal V_{init} values correspond to a thrust level of approximately 6% of maximum available thrust for the light mass case, and 7% for the heavy mass case. An aircraft trajectory as described in the previous section is computed for each of the 200×200 initial conditions in the $(\delta_{\text{fin}}, V_{\text{init}})$-plane. Various data is recorded and represented by grayscale maps over an appropriate range. Figure 3 shows the geometrical measures R and L over the grid of $(\delta_{\text{fin}}, V_{\text{init}})$-values in panels (a) and (b), respectively. For each value of δ_{fin}, simulations are performed at discrete values of V_{init} increasing from $V_{\text{init}} = 5$ m/s to $V_{\text{init}} = 25$ m/s and points at which there is a transition from stable solutions to laterally unstable solutions are detected. Specifically, if the lateral velocity of the aircraft exceeds 5 m/s then this indicates that lateral stability has been lost. This choice of lateral velocity is consistent with a previous study by the authors [11] as a value for which the aircraft has been subject to a loss of lateral stability. The transition occurs along the black curve in each of the panels in Fig. 3; white points that lie above this curve correspond to laterally unstable turns. Figure 3a shows that the turn radius R decreases with an increase in δ_{fin}. Note that R is independent of V_{init}, which is consistent with kinematic turning at low speeds [15], and follows from the fact that R is a measure of the stable turning circle solution to which the trajectories converge. Furthermore, the small changes in thrust used to set V_{init} do not affect R. However, V_{init} has a significant effect on the transient behavior before convergence to a stable turning circle. This is reflected in Fig. 3b, which shows that the approach lag L increases with V_{init}. Recall that L increases with V_{init} because, with a greater initial velocity, the aircraft will travel further before executing the turn. There is also an increase in L with δ_{fin} because the steering rate is limited; it takes longer for the steering ramp to reach the target steering angle with increased δ_{fin}.

III. Operating Region for Typical Taxiway Turns

In this section operating regions are identified for different types of turning maneuver. The aim is to define the regions such that they represent a range of possible ways in which the different maneuvers are performed. The first step is to relate the parametrized turn described in Sec. II to specific turning maneuvers. Typical taxiway geometries are chosen that are representative for the turning maneuver under consideration. In Sec. III.A bounds are identified that restrict the study to parameter values for which the aircraft follows a trajectory suitable for the specific taxiway geometry. These bounds ensure that the operating region only consists of parameter values for which the aircraft remains safely within the taxiway geometry and does not excessively overshoot the turn. The second step is to ensure that the parameter values in the operating region do not exceed other criteria for practical turns. In Sec. I it was reasoned that the maximal lateral load factors at CG reported in the FAA studies in [1] are a practical upper bound for the operation of the aircraft. The criterion chosen in Sec. III.B is that the lateral load factor during the turn does not exceed the values in the FAA studies for the different types of maneuver.

A. Relating Parametrized Turn Trajectories to Specific Maneuvers

A general method is described that relates output trajectories of the parametrized turn directly to maneuvers performed while exiting the runway and moving between taxiways. Each trajectory output is effectively fitted to the taxiway geometry upon which the maneuver is performed. The initial point in the trajectory is aligned to the entrance vector of the turn and the point on the trajectory at which the aircraft has rotated sufficiently to complete the turn is aligned with the exit vector of the turn. This works on the reasonable assumption that the steering is applied by the pilot at the appropriate distance from the turn entrance. Furthermore, it is assumed here that, if the end point of the aircraft's trajectory is approximately tangential to the exit vector of the turn, then it is possible to straighten out the aircraft to exit the turn. In this way, the data from a single computation at a specific value of δ_{fin} and V_{init} can be related to any turn geometry.

Two types of turning maneuver are considered: a shallow runway turnoff maneuver and a taxiway-to-taxiway transition. For simplicity the single taxiway geometry of a 45° turn at a group V category

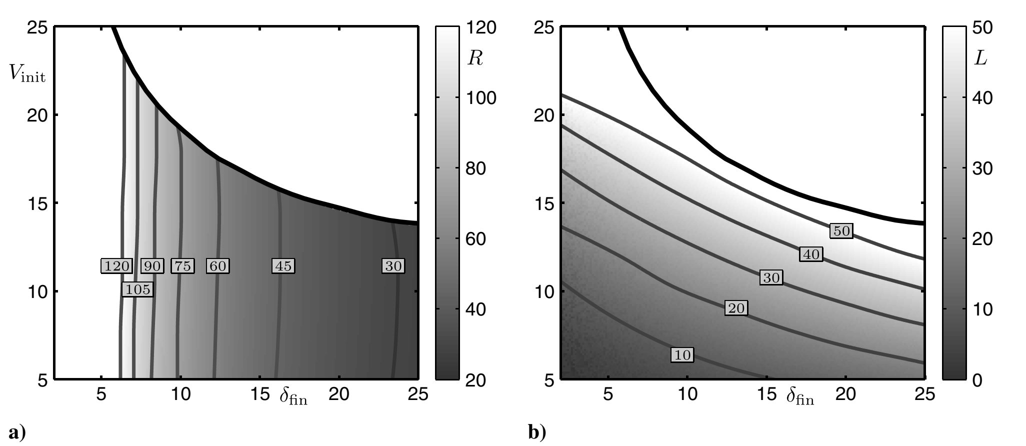

Fig. 3 Grayscale maps showing values of a) turn radius R and b) approach lag L over the shown range of V_{init} and δ_{fin} values; contours are plotted in gray. The thick black curve is the limit of stable turning; white points above it correspond to laterally unstable turns.

airport [16] is taken to be representative of a shallow runway turnoff maneuver. The group V category is chosen because it is the standard airport geometry for which maneuverability studies are performed. Secondly, a 90° turn at a group V category airport is taken to be representative of the taxiway-to-taxiway transition. To allow for direct comparison the turn radius is 45 m for both geometries. The method described above is now demonstrated by relating a single output trajectory to two different turning maneuvers. Figure 4a shows the output trajectory of the parametrized turn for $(\delta_{\text{fin}}, V_{\text{init}}) = (10°, 12\ \text{m/s})$ plotted over the (X, Y) ground plane; the aircraft has turned through 360° at the end of the trajectory. Panels (b) and (c) show the geometry for a 45 and 90° turn, respectively. The taxiway limits are plotted as solid gray curves and the turn centerlines, straight sections of which correspond to the entrance and exit vectors of the turn, are plotted as dashed gray curves. In each case a section of the parametrized turn is plotted over the taxiway geometry; the trajectories end when the aircraft has turned through 45 or 90°, as appropriate. Panels (b) and (c) illustrate how the same parametrized turn trajectory can be related to the two different turn geometries. For the same values of $(\delta_{\text{fin}}, V_{\text{init}}) = (10°, 12\ \text{m/s})$, the parametrized turn corresponds to following the turn centerline closely for the 45° turn and the ILG almost exiting the taxiway for the 90° turn.

The pilot can ensure that the ILG remains a safe distance from the edge of the taxiway by following the turn centerline with either the NLG or the approximate aircraft CG position. The former approach of following the turn centerline (painted on the taxiway) with the NLG is used for shallow turns such as the 45° turn considered here. The NLG is approximately at the same position as the cockpit and, for high speed turns, following the turn centerline with the NLG ensures that the pilot does not overshoot the turn. Therefore, in order to study the 45° turn, the maximum deviation of the NLG from the turn centerline is denoted D_{NLG}; for $(\delta_{\text{fin}}, V_{\text{init}}) = (10°, 12\ \text{m/s})$ the NLG slightly undershoots the turn and $D_{\text{NLG}} \approx 2.5$ m. For a 90° turn the pilot aims to follow the turn centerline with the approximate position of the aircraft's CG; this ensures that the ILG does not come close to the edge of the taxiway even for a tight turn. Therefore, in order to study the 90° turn, the maximum deviation of the CG position from the turn centerline is denoted D_{CG}; for $(\delta_{\text{fin}}, V_{\text{init}}) = (10°, 12\ \text{m/s})$ the aircraft significantly undershoots the turn and $D_{\text{CG}} \approx 12.5$ m. The aircraft should operate such that all landing gears are at least 4.5 m from the edge of the taxiway as specified by the design of the taxiway geometry [16]; here this is relaxed to 3 m in order to capture turns that marginally exceed the safety limit. In the trajectory shown in Fig. 4c the ILG comes within 3 m of the edge of the taxiway. The two properties D_{NLG} and D_{CG} are used to determine bounds that identify suitable trajectories in the $(\delta_{\text{fin}}, V_{\text{init}})$-plane. Specifically, a left-hand bound on δ_{fin} and V_{init} ensures that the ILG does not come too close to the edge of the taxiway. A right-hand bound on δ_{fin} and V_{init} ensures that the aircraft does not overshoot the turn centerline (with the NLG in the 45° turn or the CG in the 90° turn). An excessive overshoot of the centerline is prohibited as this corresponds to the aircraft following a turn of unnecessarily small radius. Although the quantities D_{NLG} and D_{CG} are closely related, it is convenient to consider them separately for the two different turns.

Figures 5a and 5b show grayscale maps of D_{NLG} for the 45° turn and D_{CG} for the 90° turn, respectively. Contours of D_{NLG} and D_{CG} are plotted as dashed gray curves. Contours to the left that are labeled with an underlined value represent an undershoot of the turn centerline. Similarly, contours to the right that are labeled with a bar over the value represent an overshoot of the turn centerline. In Fig. 5a there is a dark central region bounded by the curves $D_{\text{NLG}} = \underline{1}$ m and $D_{\text{NLG}} = \bar{1}$ m that represents the trajectories for which the NLG closely follows the turn centerline (within ± 1 m). Similarly, in Fig. 5b the region bounded by the curves $D_{\text{CG}} = \underline{1}$ m and $D_{\text{CG}} = \bar{1}$ m represents the trajectories for which the CG position closely follows the turn centerline (within ± 1 m). The shading gets lighter to the left of the central region representing a greater undershoot and lighter to the right of the central region indicating a greater overshoot. Note that away from the central region the contours are closer together for the 90° turn because the aircraft must follow the turn centerline for longer. The operational limits for the two turn cases are defined in terms of δ_{fin} and V_{init} by identifying specific contours in Fig. 5. For the 45° turn the contour $D_{\text{NLG}} = \underline{1}2$ m provides the left-hand bound, which ensures that the ILG remains at least 3 m from the edge of the taxiway. The contour $D_{\text{NLG}} = \bar{1}$ m provides the right-hand bound, which ensures that the aircraft does not excessively overshoot the turn centerline. Similarly, the bounds for the 90° turn are defined as $D_{\text{CG}} = \underline{1}2$ m and $D_{\text{CG}} = \bar{1}$ m. Again, these bounds ensure that the ILG remains at least 3 m from the edge of the taxiway and the aircraft does not excessively overshoot the turn centerline. From a practical point of view the undershoot criteria are more important. The bounds identified here are used to define an operational region in terms of δ_{fin} and V_{init} in Sec. III.B.

B. Maximal Lateral Loading Conditions and Operating Region

In this section values of the parameters δ_{fin} and V_{init} for which the aircraft experiences the limiting lateral load factors reported in [1] are identified. Recall from Sec. I that the maximal lateral load factor recorded for the aircraft under consideration is 0.25 g during the runway turnoff phase and 0.2 g during the taxi phase. Therefore, the aim here is to determine values of the parameters δ_{fin} and V_{init} for which lateral load factor generated is 0.25 g for a 45° turn and 0.2 g for a 90° turn. This information describes an upper bound on the operation of the aircraft during taxiing for the two types of turn.

Figure 6 shows a grayscale map of the lateral load factor N_{CG} for the trajectory represented by each $(\delta_{\text{fin}}, V_{\text{init}})$-pair; dashed white curves show contours of N_{CG}. The figure shows that for turns performed at high velocity, the lateral load factor increases rapidly with increased steering angle. Conversely, at lower velocities N_{CG}

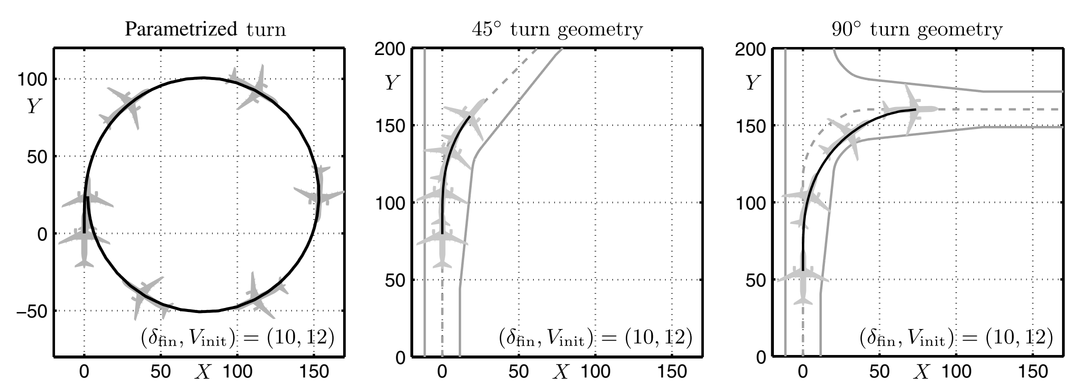

Fig. 4 Parametrized turn to turning maneuver: a) trace of the aircraft's CG (black curve) for the parametrized turn at $(\delta_{\text{fin}}, V_{\text{init}}) = (10°, 12\ \text{m/s})$ plotted over the (X, Y)-ground plane (the aircraft has turned through 360° at the end of the trajectory) and b–c) taxiway geometry for 45 and 90° turns with the taxiway limits plotted as solid gray curves and the turn centerlines plotted as dashed gray curves. In each case the respective section of the parametrized turn is plotted; the trajectories end when the aircraft has turned through 45 and 90° in panels (b) and (c), respectively.

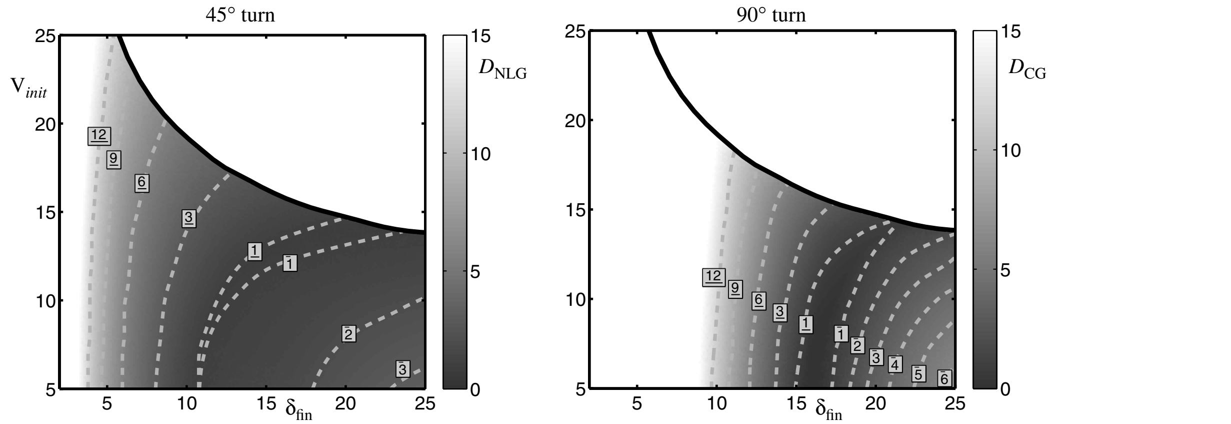

Fig. 5 Grayscale maps showing a) D_{NLG} for the 45° turn and b) D_{CG} for the 90° turn. Contours of D_{NLG} and D_{CG} plotted as gray curves; contours labeled with an underlined value represent an undershoot and contours labeled with a bar over the value represent an overshoot.

increases slowly with increased steering angle. The lateral stability boundary appears to coincide with a limit lateral load factor of approximately 0.35 g. Note that for all the aircraft considered in [1,2,4] the lateral load factor does not exceed 0.35 g. With increasing N_{CG} the contours bound a larger region; this property that N_{CG} increases as the lateral stability boundary is approached is important. In general, for any aircraft, an increase in the lateral load factor with increased V_{init} or δ_{fin} is expected: when following a steady turning circle then $N_{CG} \propto \frac{V^2}{R}$ (or approximately, $N_{CG} \propto V^2 \times \delta$), where V is the aircraft's velocity and R is the radius of the turning circle corresponding to the steering angle δ.

From the maximal lateral load values in the FAA studies it is inferred that for the runway turnoff maneuver the aircraft's operation corresponds to values of δ_{fin} and V_{init} below the 0.25 g contour. Similarly, for taxiway-to-taxiway transitions the aircraft's operation corresponds to values of δ_{fin} and V_{init} below the 0.2 g contour. This information is used in conjunction with the bounds specified in terms of D_{NLG} and D_{CG} to define operating regions for the two types of turn.

Figures 7a and 7b show the resulting operating regions for the 45 and 90° turns, respectively. The left-hand limits of the operating regions show that for increasing degree of turn, a larger δ_{fin} is required to keep the ILG a suitable distance from the edge of the taxiway; compare $D_{NLG} = \underline{1}2$ in panel (a) with $D_{CG} = \underline{1}2$ in panel (b). Again, the right-hand limit occurs at higher values of δ_{fin} with increased degree of turn. A larger steering angle is required for the NLG or CG position to follow the turn centerline and, for the 90° turn, the aircraft must follow the centerline for longer; compare $D_{NLG} = \bar{1}$ in panel (a) with $D_{CG} = \bar{1}$ in panel (b). For the 45° turn, the bound on N_{CG} is at larger values of δ_{fin} and V_{init} and closer to the lateral stability boundary; compare $N_{CG} = 0.25$ in panel (a) with $N_{CG} = 0.2$ in panel (b). Because of the larger velocities associated with the runway turnoff maneuver (45° turn), the corresponding lateral load factor is larger than during taxiway-to-taxiway transitions (90° turn). The operating regions are plotted over a grayscale map of the lateral NLG load N_{NLG}. It is convenient to show this information because the maximal landing gear loads across each operating region are studied in Sec. IV. The lateral NLG load N_{NLG} is chosen because the NLG experiences the most critical lateral loading during turning. Note that the values of N_{NLG} are independent of the degree of the turn because the maximal loads on the NLG occur while the steering is being ramped up to δ_{fin}; this happens before the aircraft has turned through 45° (the same holds for the ILG and OLG within the operating regions). The operating region for the 45° turn encompasses values of δ_{fin} and V_{init} corresponding to values of N_{NLG} that are close to the regulation's limit of $N_{NLG} = 0.5$. An important feature of the data shown in Fig. 7 is that for both operating regions N_{NLG} is uniformly increasing as δ_{fin} and V_{init} approach the N_{CG} boundary. This property also holds for lateral ILG and OLG loads. Therefore, to find the maximal lateral gear loads in a given operating region it is sufficient to study the loads solely along the N_{CG} boundary.

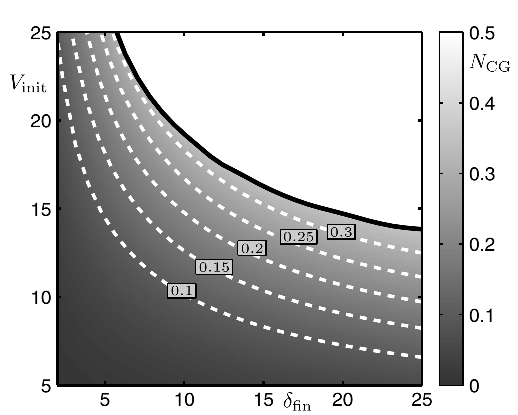

Fig. 6 Grayscale map of the maximal lateral load factor N_{CG} for the output trajectory initialized from each (δ_{fin}, V_{init})-pair; dashed white curves are contours of N_{CG}.

IV. Maximal Lateral Gear Loads in Operating Regions

Since the maximal lateral gear loads in each operating region are attained at the N_{CG} boundary, the N_{CG} curve is parametrized in order to get a representation of the maximal lateral gear loads in the operating regions depending on δ_{fin}. Effectively the problem of finding the limiting loads has been reduced to computing these values along a one-dimensional curve. Given that the criteria for defining a region of standard operations can be applied to any aircraft configuration, the limiting loads are computed for light and heavy aircraft cases. For both mass cases and the two types of taxiway turn, the lateral gear load values are found along the corresponding N_{CG} boundary at 50 discrete values of δ_{fin}. In this way, the lateral gear load values are extracted along the operating limit curves in Fig. 7.

Figure 8 shows plots of the lateral gear loads N_{NLG}, N_{ILG} and N_{OLG} recorded along the N_{CG} limit parametrized in terms of δ_{fin}; the lateral load factor N_{CG} is also plotted at its constant value. The top two panels (a) and (b) represent the light aircraft case for which the operating regions are shown in Fig. 7; the bottom panels represent the heavy aircraft case. The first column corresponds to the 45° turn and the second column to the 90° turn. In each panel of Fig. 8 vertical black lines indicate the δ_{fin} value corresponding to the lower extent of the N_{CG} curve; the section shaded gray represents the values of δ_{fin} corresponding to the appropriate operating region. The limits of the gray region correspond to intersections between N_{CG} and the appropriate D_{NLG} and D_{CG} curves.

First, the distribution of lateral loads between the ILG and the OLG is discussed. The lateral ILG and OLG loads are closely related to the lateral load factor N_{CG}: N_{ILG} and N_{OLG} vary linearly with δ_{fin} in all panels of Fig. 8. Panels (a) and (b) show that, in the light case, for both types of turn, N_{OLG} is larger than N_{ILG}. Within the operating

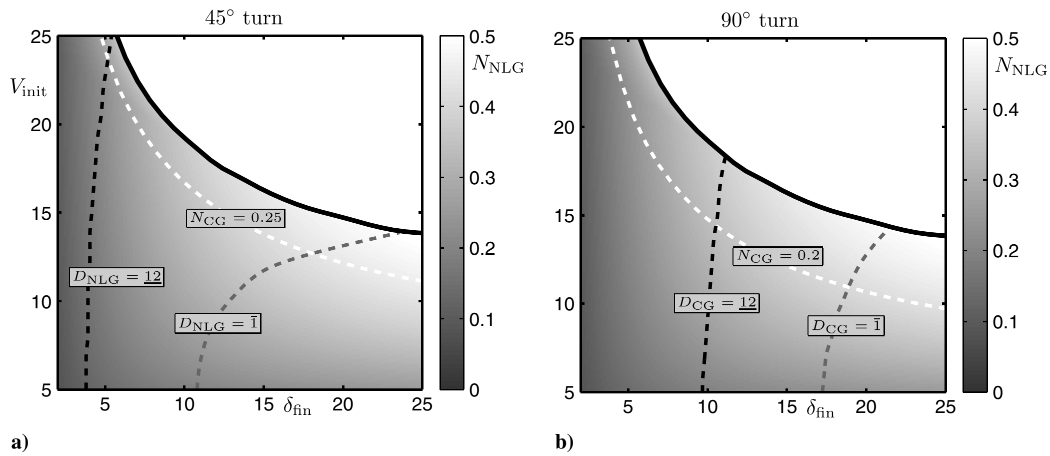

Fig. 7 Grayscale maps show the lateral NLG load N_{NLG} for the trajectory represented by each (δ_{fin}, V_{init})-pair. The operating regions are represented by the values of δ_{fin} and V_{init} that lie inside bounds on D_{NLG}, D_{CG} and N_{CG} as shown.

region for the 45° turn N_{OLG} is at most 40% larger than N_{ILG}; for the 90° turn the difference is at most 20% larger. This difference can be accounted for by the fact that during a turn the aircraft's weight shifts to the outside gear and the OLG takes a larger vertical load; in general the lateral load generated by a tire increases with vertical load. In the heavy aircraft case, for both types of turn, there is a value of δ_{fin} above which N_{ILG} is larger than N_{OLG}; see panels (c) and (d). Because of the aircraft geometry the ILG generates a larger slip angle while turning. For stable turns the lateral forces generated by the tires increase with slip angle and in the heavy case there is some value of δ_{fin} for which this effect dominates over the larger vertical load at the OLG. For a heavy aircraft in the operating region for the 45° turn $N_{\text{OLG}} > N_{\text{ILG}}$ with the values becoming equal at the maximal value of δ_{fin}; see panel (c). Conversely, in the operating region for the 90° turn, $N_{\text{ILG}} > N_{\text{OLG}}$ with the values being equal at the minimal value of δ_{fin}; see panel (d). Across all four cases shown in Fig. 8, the lateral load factor is a good predictor of the lateral ILG and OLG loads. Furthermore, N_{ILG} and N_{OLG} are less than or equal to the lateral load factor at CG (with a slight exception for the OLG in Fig. 8a).

Across all four cases shown in Fig. 8 the lateral NLG loads N_{NLG} are greater than N_{OLG}, N_{ILG} and N_{CG}. The loads at the NLG increase with δ_{fin} and the maximal values occur at the upper limit of δ_{fin}. In the operating regions for the 45° turn N_{NLG} is approximately equal to N_{CG} for small values of δ_{fin}; see panels (a) and (c). However, as δ_{fin} increases there is a rapid deviation and the lateral NLG load is vastly underestimated by the lateral load factor at CG. Furthermore, at the upper limit of δ_{fin} the loads at the NLG come close to $N_{\text{NLG}} = 0.5$, which is the limit imposed by the FAA. For all values of δ_{fin} in the operating regions for the 90° turn N_{NLG} is vastly underestimated by N_{CG}; see panels (b) and (d). At the upper limit of δ_{fin} the loads at the NLG are underestimated by a factor of two. In conclusion, studying the lateral load factor at CG alone is insufficient for the prediction of

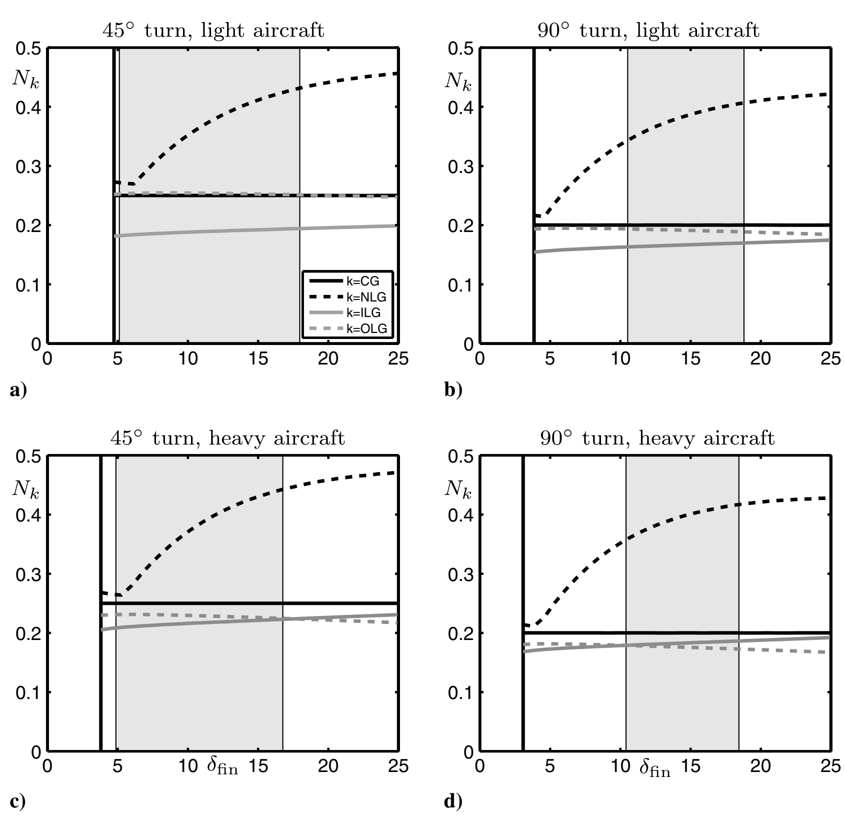

Fig. 8 Lateral loads N_{CG}, N_{NLG}, N_{ILG} and N_{OLG} computed along the N_{CG} boundary curves and parametrized in terms of δ_{fin}; turn degree and mass case is indicated at the top of each panel. In each panel the region shaded in gray represents the values of δ_{fin} corresponding to the appropriate operating region as shown in Fig. 7. A vertical black line indicates the lower extent of the parametrized N_{CG} curve.

the loads at the landing gears. Note that the large change in mass between the light and heavy cases corresponds to only a marginally increased lateral NLG load. The largest loads at the OLG occur for the light mass case.

V. Operating Region for Efficient Turns

In Secs. III and IV the upper limit of operation was defined in terms of the maximal lateral load factors shown in Fig. 1. In this way, the limits of the operating regions represent the extremes of the aircraft's operation. However, the approach presented in this paper is very flexible and other limits can be defined in a similar way with any relevant criteria that provide a bound within which it is desirable for the aircraft to operate. As an example, operating regions are now defined in terms of a target for the efficiency of turns. Specifically, a turn can be considered efficient if during the turn a large proportion of the approach velocity is conserved.

The velocity lost during a turn, V_{loss}, is expressed as a percentage by the equation

$$V_{\text{loss}} = 100 \times \frac{V_{\text{init}} - V_{\text{fin}}}{V_{\text{init}}}$$

where V_{fin} is the velocity of the aircraft when it reaches the exit vector of the turn. For smaller values of V_{loss} less velocity is lost and the turn is more efficient. Figure 9 shows grayscale maps of V_{loss} for the two types of turn; contours of V_{loss} are plotted as white curves. The plots show that more velocity is lost with a higher-degree turn. Specifically, panel (a) shows that for the 45° turn the maximal value of V_{loss} just exceeds 10% in the stable region; the largest value occurs close to the lateral stability boundary at high δ_{fin}. For the 90% turn, see panel (b), the maximal value of V_{loss} just exceeds 18% with the maximal values occurring close to the lateral stability boundary. The relative spacing between the contours for the two types of turn shows that there is a larger penalty in terms of efficiency when increasing δ_{fin} and V_{init} for the 90% turn. Therefore, depending on the type of turn, different V_{loss} limits are chosen as the criteria for suitably efficient maneuvers.

Contours of V_{loss} are now specified to represent upper limits for new operating regions that, as in Sec. III.B, take into account appropriate limits for D_{NLG} and D_{CG}. An upper limit of $V_{\text{loss}} = 4\%$ is taken for the 45° turn and $V_{\text{loss}} = 8\%$ for the 90° turn. These limits are chosen such that in these new operating regions the lateral load factor does not exceed the maximal values identified in the FAA studies for the light aircraft case. Accordingly, $N_{\text{CG}} < 0.25$ g along $V_{\text{loss}} = 4\%$ for the 45° turn, and $N_{\text{CG}} < 0.2$ g along $V_{\text{loss}} = 8\%$ for the 90° turn. The same V_{loss} limits are chosen for the heavy aircraft case to allow for direct comparison between the mass cases. Figure 10 shows the resulting operating regions, again plotted over a grayscale map of N_{NLG}. The new operating regions represent a subset of those defined in Sec. III.B due to the way in which the V_{loss} bounds are chosen.

The lateral load factor and lateral gear loads increase as the V_{loss} limit is approached and, therefore, in order to identify the maximal loads in the region, the loads along the V_{loss} curves are extracted. Plots of the lateral load factor and lateral gear loads are shown for

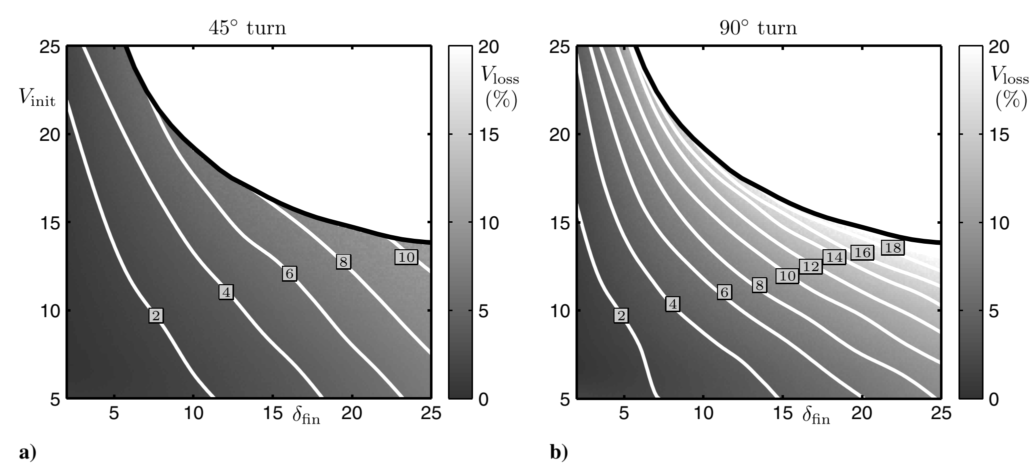

Fig. 9 Grayscale maps showing the percentage of velocity lost V_{loss} for the two types of turn for the trajectory represented by each (δ_{fin}, V_{init})-pair. White curves are contours of V_{loss}.

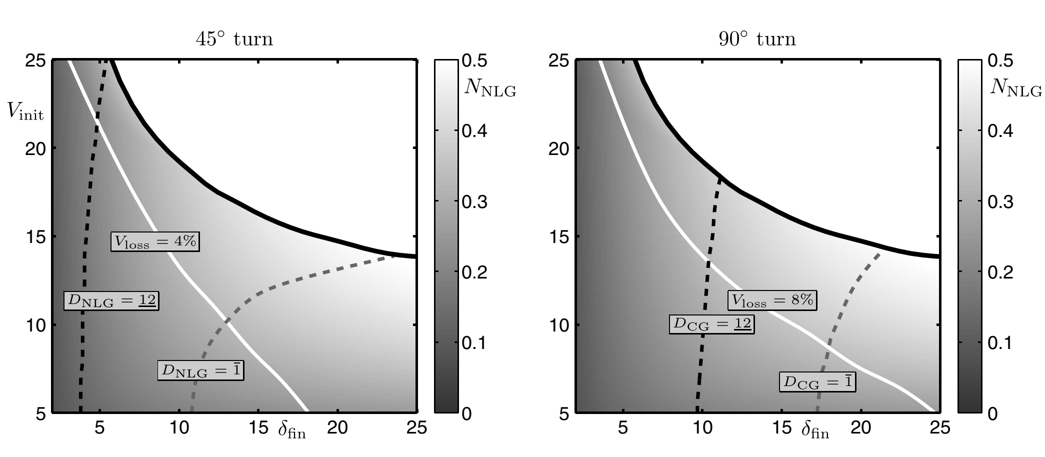

Fig. 10 Grayscale maps showing the lateral NLG load N_{NLG} for the trajectory represented by each (δ_{fin}, V_{init})-pair. The operating regions are the values of δ_{fin} and V_{init} that lie inside bounds on D_{NLG}, D_{CG} and V_{loss} as shown.

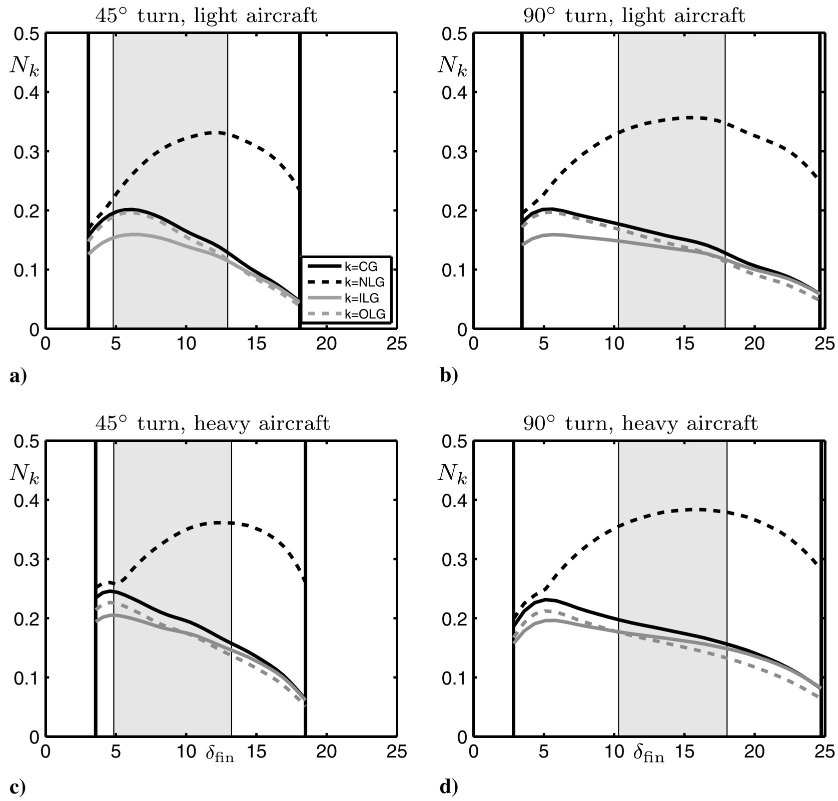

Fig. 11 Lateral loads N_{CG}, N_{NLG}, N_{ILG} and N_{OLG} computed along the V_{loss} boundary curves parametrized in terms of δ_{fin}; turn degree and mass case is indicated at the top of each panel. In each panel the gray shaded region represents the values of δ_{fin} corresponding to the appropriate operating region as shown in Fig. 10; vertical black lines indicate the upper and lower extent of δ_{fin} for the parametrized V_{loss} curve.

the two types of turn and two mass cases in Fig. 11. For the 45° turn, the lateral load factor N_{CG} peaks close to the lower extent of the operating region and steadily drops off as δ_{fin} increases; see panels (a) and (c). For the light case $N_{OLG} > N_{ILG}$ with the loads becoming equal at the upper extent of the operating region; see panel (a). For the heavy case there is a transition from the greater load being the OLG to the ILG with $N_{ILG} = N_{OLG}$ at $\delta_{fin} \approx 10°$; see panel (c). For the 90° turn, the lateral load factor decreases with increased δ_{fin} in the operating region; see panels (b) and (d). For the light aircraft case $N_{OLG} > N_{ILG}$ with the loads becoming equal at the upper extent of the operating region; see panel (b). For the heavy case $N_{ILG} > N_{OLG}$ and the loads are equal at the lower extent of the operating region; see panel (d). For both turn types with the light case, N_{CG} matches N_{OLG} very closely and is a good predictor of the loads at the main landing gears; see panels (a) and (b). For the heavy case the main lateral gear loads are more evenly distributed; see panels (c) and (d). Again, in all cases N_{CG} is a good predictor for the loads at the main landing gears. However, the loads at the NLG are vastly underestimated by N_{CG}. The main qualitative difference between the profile of N_{NLG} when compared with Fig. 8 is that with increasing δ_{fin} there is a peak value after which the load drops off; for the 45° turn this occurs at $\delta_{fin} \approx 12°$ and for the 90° turn at $\delta_{fin} \approx 16°$, independently of the mass case. The data shows that the inadequacy of N_{CG} in predicting N_{NLG} is not limited to the extremes of the aircraft's operation.

VI. Conclusions

A general approach was presented to evaluate an aircraft's performance across an entire operating region for specific turning maneuvers. The dynamic model of a tricycle-gear passenger aircraft used in this paper was fully validated against an industry-tested model in a previous study. A turn that represents pilot practice during taxiway maneuvers was parametrized in terms of approach velocity and steering input. The output trajectories of the parametrized turn were then related directly to turning maneuvers. Representative runway and taxiway geometries were chosen for two types of turning maneuver: a runway turnoff of 45° and a taxiway-to-taxiway transition of 90°. Operating regions were defined to represent a range of possible ways in which the different maneuvers are performed where the limits of the regions represent the extremes of the aircraft's operation. Specifically, the extremes of operation were taken to be the maximal lateral load factors reported in studies of in-service aircraft carried out by the FAA. Such operating regions were defined for the two types of maneuver and for two mass cases. The performance of individual landing gears was assessed across the operating regions in terms of the actual lateral loads experienced. In particular, the maximal lateral loads at the limits of the operating regions were determined.

The results show that the lateral load factor at CG is a good quantitative predictor of the loads experienced at the main landing gears; this can be attributed to the proximity of the main gears to the CG position. Asymmetric lateral loading between the main gears was investigated and it was found that whether greater loads occur at the inner or outer gear depends on the turn type and the aircraft mass. More significantly, the lateral loads at the NLG are found to be vastly underestimated by the lateral load factor at CG, in the worst case by a factor of two. In light of the large separation between the nose gear and the CG position, it may not be so surprising that the loads at the nose gear are underestimated. However, the extent of the underestimation is not clear a priori, but has been quantified as substantial in this paper. Further, note that the difference in loading between main and NLG is not actually taken into account in the existing regulation. Another finding is that, for the same lateral load factor at CG, the actual lateral gear loads are largely unaffected by changes to the aircraft weight. Overall, the results show that any investigation into lateral loading during taxiing operations should not be limited to studying the lateral load factor at CG. Furthermore, should future studies be carried out with instrumentation of the individual landing gears, it is of paramount importance that the nose gear be included. In conjunction with existing studies the results presented here suggest that, for the particular aircraft under consideration, the limit imposed in the FAR is too conservative for the main landing gears. Furthermore, it may necessary to consider a separate regulation for lateral loads on the nose gear during turning.

To illustrate the generality of the approach described above, it was adapted to study lateral gear loads in operating regions based on a criterion for the proportion of the aircraft's approach velocity that is conserved during a turn. For operating regions based on the velocity conservation condition, as was shown for the lateral loading condition, it was found that the lateral load factor at CG is a good predictor of loads at the main gears, but a bad predictor of the loads at the nose gear. The robustness in the qualitative behavior also shows that the overall result is not limited to the extremes of the aircraft's operation. The velocity criterion under consideration could easily be adapted to satisfy a specific safety margin for the landing gear loads with respect to regulation limits. Such a criterion could then be implemented through pilot practice or in an automatic control system. The approach presented here is suitable for the study of any reasonable criteria on the aircraft's operation; for example, speed limits depending on taxiway conditions, limiting vertical or lateral load on a specific gear, the maximal slip angles generated at the tires, or a bound on the energy lost during maneuvers. The approach could also be used to compare the performance of aircraft of different size or investigate other changes in design. Of particular interest would be to assess the performance of different landing gear configurations and different tire characteristics, especially for heavy aircraft with more than two main gears. The focus here, as in the existing regulation, is stable turning during standard ground maneuvers, but the approach could also be used to investigate an emergency situation such as obstacle avoidance.

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